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On Ternary Quadratic Equation $x(x + y) = z + 20$

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ABSTRACT: The quadratic equation with three unknowns given by $x(x + y) = z + 20$ is analysed for its non-zero distinct integer solutions. Given a solution, formula for generating sequence of solutions is obtained. A few interesting relations among the solutions are also exhibited.

KEYWORDS: Ternary quadratic, Integer solutions.

I. INTRODUCTION

The ternary quadratic Diophantine equations offer an unlimited field for research due to their variety [1, 23]. For an extensive review of various problems, one may refer [2-22]. This communication concerns with yet another interesting ternary quadratic equation $x(x + y) = z + 20$ representing a cone for determining its infinitely many non-zero integral points. Also, a few interesting relations among the solutions are presented.

II. METHOD OF ANALYSIS

The ternary quadratic equation to be solved is

$$x(x + y) = z + 20 \tag{1}$$

Treating (1) as a quadratic in x and solving for x, we get

$$x = \frac{1}{2} \left[-y \pm \sqrt{y^2 + 4z + 80} \right] \tag{2}$$

Substituting $y = z + 19$

(3)

in (2), it is seen that $x = 1, -(z + 20)$

Thus, the triples $(1, z + 19, z)$ and $(-z - 20, z + 19, z)$ satisfy (1)

Also, the substitution of $y = 2k + 8, z = 4k$

(4)

in (2) gives $x = 2, -(2k + 10)$

∴ The triples $(2, 2k + 8, 4k)$ and $(-2k - 10, 2k + 8, 4k)$ satisfy (1)

It is noted that, (1) is not altered on replacing x by -x and y by -y simultaneously. Hence, the triples $(-1, -z - 19, z), (z + 20, -z - 19, z), (-2, -2k - 8, 4k)$ and $(2k + 10, -2k - 8, 4k)$ also satisfy (1).

However, we have other choices of integer solutions to (1) which are presented below in Table1:

Table 1: Solutions

S. No	x	y	z
1	2	k + 8	2k
2	k + 10	-k - 8	2k
3	1	2k + 19	2k
4	-2k - 20	2k + 19	2k
5	-k - 10	k + 8	2k
6	2	k + 8	4k

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7	$k + 10$	$-k - 8$	$4k$
8	5	$k - 1$	$5k$
9	$k + 4$	$-k + 1$	$5k$
10	2	$3k + 8$	$6k$
11	$3k + 10$	$-3k - 8$	$6k$
12	4	$2k + 1$	$8k$
13	$2k + 5$	$-2k - 1$	$8k$
14	10	$k - 8$	$10k$
15	$k + 2$	$-k + 8$	$10k$
16	5	$2k - 1$	$10k$

A natural question that arises now is that, whether a general formula for obtaining a sequence of integer solutions for (1) based on its given integer solution can be found?

The answer to the question is yes and a method of obtaining the same is illustrated below:

Let (x_0, y_0, z_0) be the given integer solution to (1). Let (x_1, y_1, z_1) be the first solution of (1) where

$$x_1 = h - x_0, y_1 = h - y_0, z_1 = z_0 + h^2 \tag{5}$$

where h is any non-zero integer to be determined.

Substituting (5) in (1) and simplifying, we have

$$h = 3x_0 + y_0 \tag{6}$$

Using (6) in (5), we have

$$x_1 = 2x_0 + y_0, y_1 = 3x_0, z_1 = z_0 + (3x_0 + y_0)^2$$

Repeating the above process, the general solution (x_n, y_n, z_n) of (1) is represented by

$$x_n = \frac{1}{4} \left[(3^{n+1} + (-1)^n)x_0 + (3^n - (-1)^n)y_0 \right]$$

$$y_n = \frac{1}{4} \left[(3^{n+1} - 3(-1)^n)x_0 + (3^n + 3(-1)^n)y_0 \right]$$

$$z_n = z_0 + \left(\frac{3^{2n} - 1}{8} \right) (3x_0 + y_0)^2$$

A few interesting relations among the solutions are exhibited as follows

- $x_{n+2} + 2x_{n+1} = 4y_n + 11x_n$
- $y_{n+2} + 12y_{n+1} = 3y_n + 12x_n$
- $x_{n+2} = 2y_n + 7x_n$
- $y_{n+2} = 3y_n + 6x_n$
- $x_{n+1} = y_n + 2x_n$
- $x_{n+1} + x_{n+2} = 9x_n + 3y_n$
- $y_{n+1} = 3x_n$
- $y_{n+1} + y_{n+2} = 9x_n + 3y_n$
- $3x_{n+1} = y_{n+2}$

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III. CONCLUSION

In this paper, we have made an attempt to find all integer solutions to the ternary quadratic equation given by $x(x+y) = z + 20$. To conclude, one may consider suitable changes in the choice of the linear transformation (5) to arrive at different forms of general solution to (1).

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